Lecture 15 Perspectives of development of the modern theory of automatic control of technical systems

Synthesis of control laws in case of incomplete information on control object parameters

15. 1. Synthesis of the linear laws of control of the adaptive system by a direct method of Lyapunov.

We will consider the control object (CO) which is described by the following equation:

$$\dot{X}_p = A_p X_p + B_p U. \tag{1}$$

Here the index p means expanded system, where $X_p \in R^n,\ u \in R^1;\ \ A,B$ are real matrixes of the appropriate dimensionalities, and some of elements a_{ij}, b_{ik} $i = \overline{1,n},\ j = \overline{1,n},\ k = \overline{1,l}$ (or even all) are unknowns. There is a task of synthesis of the control system which providing stability of closed system and quality of transient process (desirable response dynamic characteristics).

We will use the adaptive approach for the solving of the task. We will set structure of the equation of model similar to structure of CO:

$$\dot{X}_m = A_m X_m + B_m U,$$

where $X_n \in R^n,\ u \in R^1$ (matches a vector of control of an object), A_m, B_m are const real matrixes of the appropriate dimensionalities. As the choice of matrixes depends on the researcher (designer), they can be selected so that the model had in advance set properties.

We will enter a deviation vector:

$$e = X_m - X_p. \tag{2}$$

The vectorial differential equation of an deviation with use of the equations of a control object and model will have the following appearance:

$$\dot{e} = A_m e + (A_m - A_p)X_p + (B_m - B_p)U.$$

Further we will enter designation:

$$A_m - A_p = \Delta A,\ B_m - B_p = \Delta B,$$

and we will receive the following differential equation of deviation (3):

$$\dot{e} = A_m e + \Delta A X_p + \Delta B U. \tag{3}$$

We will assume that elements of matrixes allow independent adjustment. We will find the law of adjustment of elements of matrixes providing a condition:

$$\lim_{t \to \infty} e(t) = 0,$$

which execution guarantees the following:

$$\lim_{t \to \infty} X_p(t) = X_m(t).$$

For finding of the law of adjustment we will consider Lyapunov's function of the following look:

$$V = e' Pe + tr\left(\Delta A' R^{-1} \Delta A\right) + tr\left(\Delta B' G^{-1} \Delta B\right), \qquad (4)$$

where matrix $P = P^T > 0$ is the symmetric, fixed-sign positive matrix which is subject to determination.

Matrixes $R^{-1} = \left(R^{-1}\right)' > 0$ *and* $G^{-1} = \left(G^{-1}\right)' > 0$ select for support of the required speed of attenuation of the deviation $e(t)$.

The total derivative of function of Lyapunov on time owing to system (3) will be following:

$$\frac{dV}{dt} = -e' Qe + 2tr\left[\Delta A'_p \left(PeX'_p - R^{-1} \dot{A}_p\right)\right] + 2tr\left[\Delta B' \left(PeU' - G^{-1} \dot{B}_p\right)\right],$$

where matrix $Q = Q' > 0$ is connected to a matrix P matrix equation of Lyapunov:

$$A'_m P + PA_m = -Q. \qquad (5)$$

As the choice of a matrix A_m of model is in the researcher's hands, having selected A_m Gurviz and having set by a matrix $Q = Q^T > 0$, it is possible to find the single solution of the matrix equation (5) concerning a matrix $P = P^T > 0$.

If process of adjustment of matrixes satisfies to the following matrix equations (6-7):

$$\dot{A}_p = RPeX'_p \qquad (6)$$

$$\dot{B}_p = GPeU', \qquad (7)$$

that for $\frac{dV}{dt}$ we will receive expression of the following look:

$$\dot{V} = -e' Qe.$$

The matrix skeleton diagram of system can be provided as follows (fig. 1):

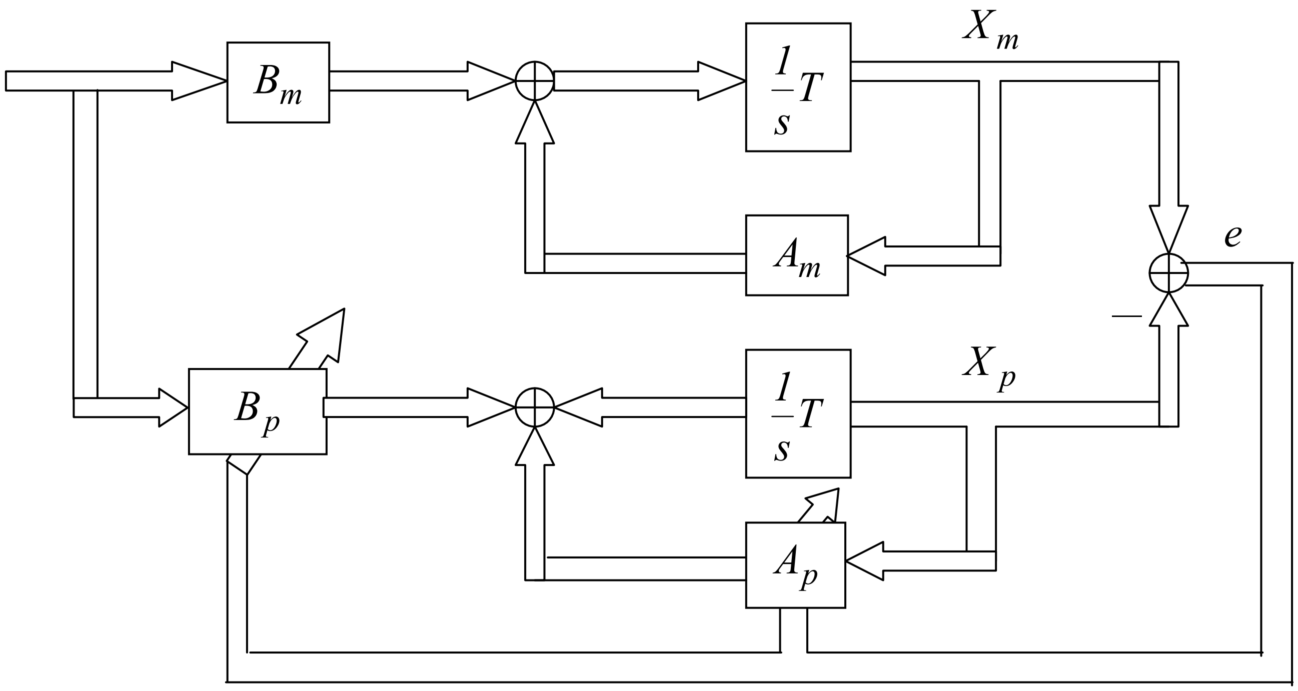

Fig. 1 – The matrix skeleton diagram of the adaptive system with adjustment

Hence, if the law of adjustment of elements of matrixes is defined by the solution of the equations (6) and (7), then the zero solution of system (3) is asymptotically steady across Lyapunov. It, in turn, provides execution of a condition $X_p(t) = X_m(t)$ by $t \to \infty$. Practically, it is never executed $e(t) = 0$ (nobody waits $t = \infty$), therefore process of adjustment comes to the end for a finite period t_1 after which the deviation $|e(t)| \le< 0$ does not exceed the given rather small value some beforehand $\delta < 0$.

Matrixes R, G, P influence dynamic characteristics of process of adjustment (it is visible from the equations (6) and (7)); therefore matrixes R, G select from conditions of support of the required adjustment speed, and matrix P (the solution of the matrix equation of Lyapunov (5)) set indirectly, setting a matrix $Q = Q' > 0$.

In case of a research of speed of adjustment of parameters of system from change of a matrix $R(r_{ii})$, a matrix Q, dependence of adjustment of parameters on control process (fig. 2).

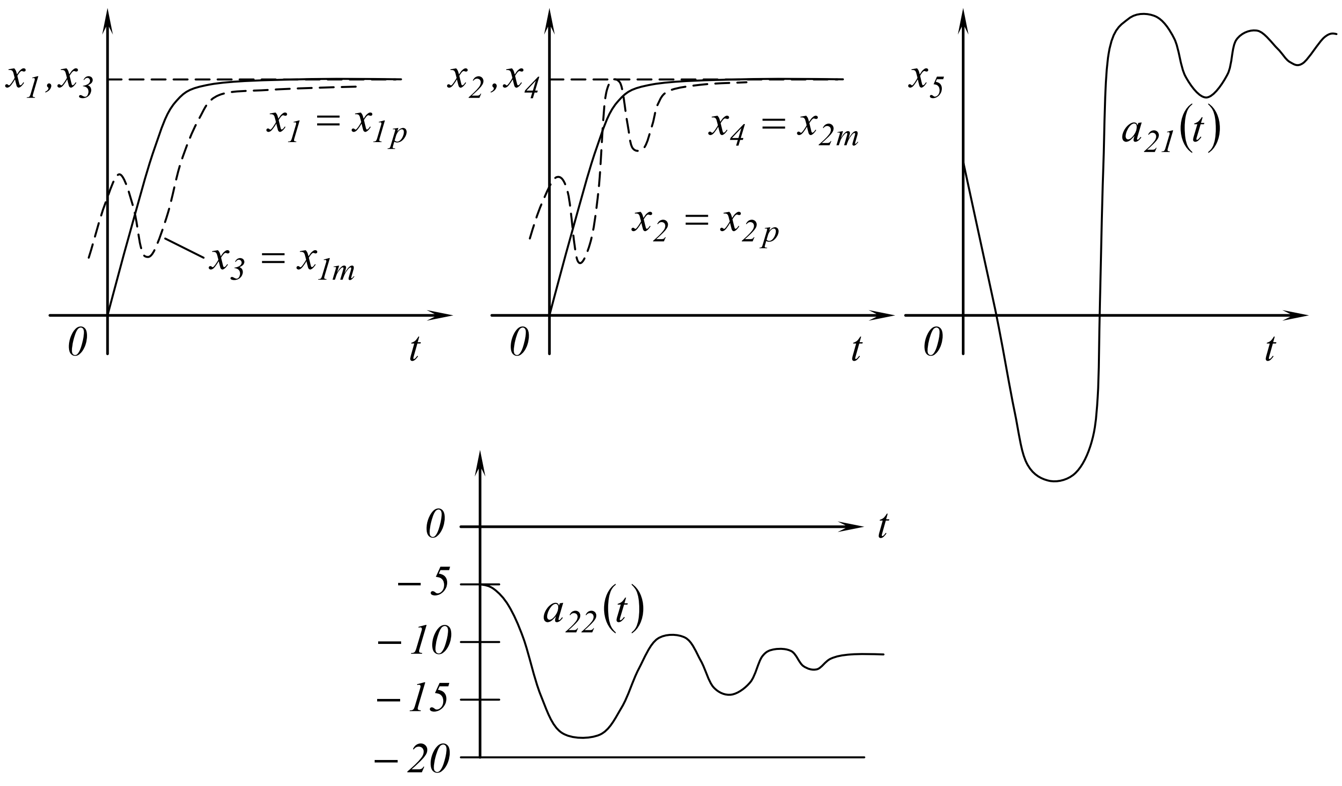

Fig. 2 – Transient process of the adaptive control system

It is visible that with increase r_{22} in a matrix $R(r_{ii})$ variability of system decreases.

15. 2. Synthesis of nonlinear laws of control

Let the following control system be set:

$$\dot{X} = AX + BX, \tag{8}$$

where $X \in R^n$, $u \in R^1$, $\quad \mathrm{Re}(Si(A)) < 0, \quad i = \overline{1, n}$.

Restrictions are imposed on components of the control vector:

$$\left| u_j(t) \right| \le M_j < \infty, \quad j = \overline{1, l},$$

which account in case of synthesis of the control law leads to nonlinear closed system. For finding of the control law of closed system we will use Lyapunov's function:

$$V = X' PX,$$

where matrix $P = P' > 0$ is the single solution of the matrix equation of Lyapunov:

$$A' P + PA = -Q$$

for arbitrary matrix $Q = Q^T > 0$. We will find a total derivative on time function

$\dfrac{dV}{dt}$ owing to system (8):

$$\dot{X} = AX + BU; \ \left(\dot{X}\right)' = A'X' + B'U'.$$
$$\dot{V} = \dot{X}'PX + X'P\dot{X} = \left(X'A + U'B'\right)PX + X'P\left(AX + BU\right) = \\ = X'\left(A'P + PA\right)X + U'B'PX + X'PBU = -X'QX + 2U'B'PX,$$

i.e. $\dot{V} = -X'QX + 2u'B'PX$.

For support of stability of closed system we will select $u_j(t) \forall \ j = \overline{1, l}$ from the following look:

$$u_j(t) = -M_j \, sign(m_i(t)), \tag{9}$$

where $m_i(t), \ i = \overline{1, l}$ there are vector components $m(t) = B^T PX(t)$, and function $sign(m_i)$ определяется как:

$$sign(m_i) = \begin{cases} +1, if \ m_i > 0 \\ \\ -1, if \ m_i < 0 \end{cases}. \tag{10}$$

The matrix skeleton diagram of the control system has the following appearance (fig. 3):

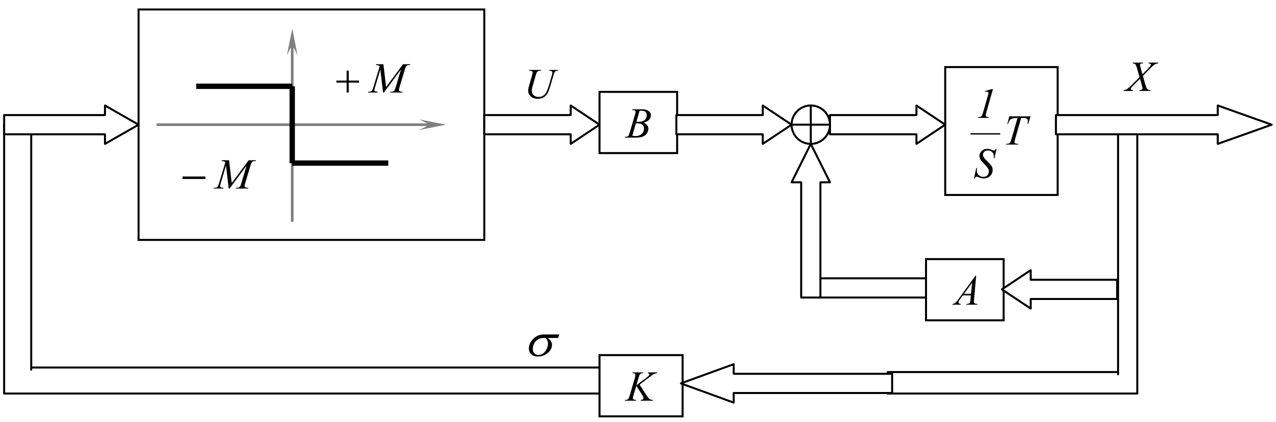

Fig. 3 – The matrix skeleton diagram of the control system with nonlinearity of type (10)

Thus, received closed control system with explosive function of control. In certain cases in the system (8) shorted by the law of control (9) there can be no decisions, i.e. living conditions of decisions can be violated. Then it is possible to approximate the found control law (9) the following function (11):

$$u_j(t) = -u_j sat(k_j m_j(t)), \qquad (11)$$

where $sat(z_j) = \begin{cases} 1, \text{ если } z_j > 1 \\ 0, \text{ если } |z_j| \le 1 \\ -1, \text{ если } z_j < -1 \end{cases}; \quad z_j = k_j m_j(t).$

Selecting k_j rather big value, it is possible to approximate $sign(z_j)$ as much as precisely function by means of function $sat(kz_j)$. However in case of such changeover in closed system the limit cycle can be generated. Nevertheless, such changeover can be justified as the limit cycle can be made as much as small way of a choice k_j rather big.